TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 1 2020

Section 1 Calculator Free

Differentiation, Applications of Differentiation, Integration, Applications of Integration

STUDENT'S NAME SOLUTIONS

DATE: Friday 6th March **TIME:** 20 minutes **MARKS:** 18

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine each of the following

(a) $\int \frac{3x-5}{x^4} dx$ [2]

$$= \int 3x^{-3} - 5x^{-4} \, dx$$

$$= \frac{3x^{-2}}{-2} - \frac{5x^{-3}}{-3} + c$$

$$= \frac{-3}{2x^2} + \frac{5}{3x^3} + c$$

(b) $\int \ 6(2x+3)^5 dx$ [3]

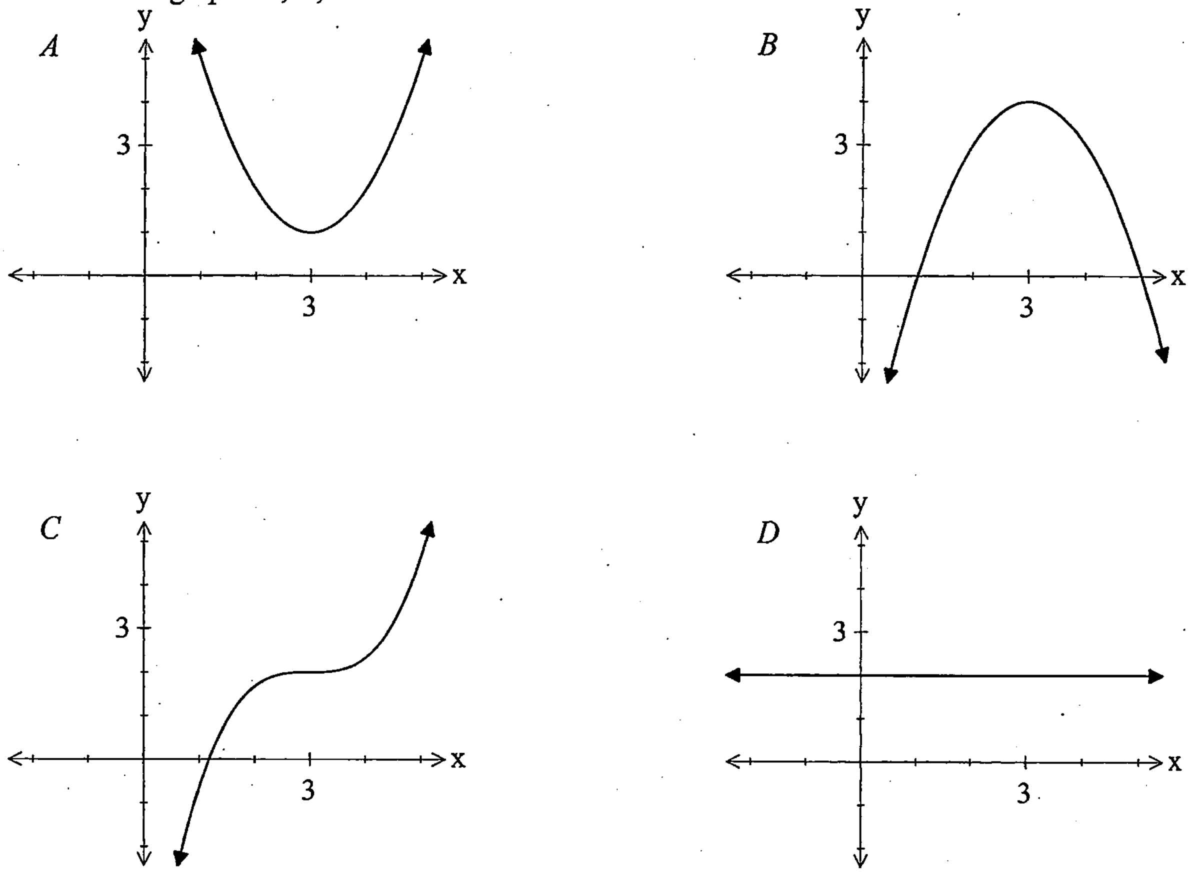
$$= 3\int 2(2x+3)^5 \, dx$$

$$= \frac{3(2x+3)^6}{6} + c$$

$$= \frac{(2x+3)^6}{2} + c$$

2. (4 marks)

Examine the graphs A, B, C and D below:



List ALL graphs in which

(a) $f'(3) = 0$ A, B, C, D

(b) $f'(4) > 0$ A, C

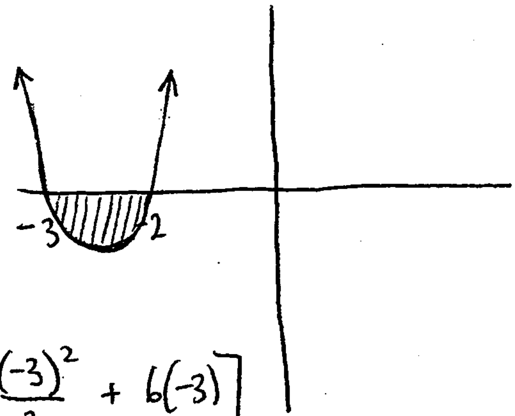
(c) $f''(3) < 0$ B

(d) $f''(4) > 0$ A, C

3. (4 marks)

Determine the area bound by the x-axis and $y = 6 + 5x + x^2$.

$y = (x+2)(x+3)$



$$= \int_{-3}^{-2} x^2 + 5x + 6 \, . \, dx$$

$$= \left[\frac{x^3}{3} + \frac{5x^2}{2} + 6x \right]_{-3}^{-2}$$

$$= \left[\frac{(-2)^3}{3} + \frac{5(-2)^2}{2} + 6(-2) \right] - \left[\frac{(-3)^3}{3} + \frac{5(-3)^2}{2} + 6(-3) \right]$$

$$= \left[\frac{-8}{3} + 10 + (-12) \right] - \left[-9 + \frac{45}{2} - 18 \right]$$

$$= \left[\frac{-8}{3} - \frac{6}{3} \right] - \left[-\frac{54}{2} + \frac{45}{2} \right]$$

$$= \frac{-14}{3} + \frac{9}{2}$$

$$= \frac{-28}{6} + \frac{27}{6}$$

$$= \frac{1}{6} \text{ units}^2$$

4. (5 marks)

Determine the equation of the curve with a minimum turning point of $(3, -5)$ and a second derivative of $12x - 18$.

$$= \int 12x - 18 \,.\, dx$$

$$y' = \frac{12x^2}{2} - 18x + c$$

$$y' = 6x^2 - 18x + c$$

$$0 = 6(3)^2 - 18(3) + c$$

$$c = 0$$

$$\therefore y' = 6x^2 - 18x.$$

$$y = \int 6x^2 - 18x \,.\, dx$$

$$= \frac{6x^3}{3} - \frac{18x^2}{2} + c$$

$$y = 2x^3 - 9x^2 + c$$

$$-5 = 2(3)^3 - 9(3)^2 + c$$

$$-5 = 54 - 81 + c$$

$$c = 22$$

$$\therefore y = 2x^3 - 9x^2 + 22$$

TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 1 2020

Section 2 Calculator Assumed

Differentiation, Applications of Differentiation, Integration, Applications of Integration

STUDENT'S NAME ______________________________

DATE: Friday 6th March **TIME:** 30 minutes **MARKS:** 33

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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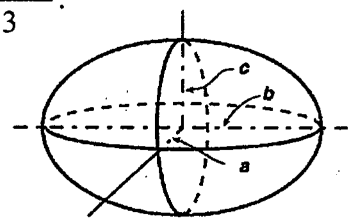
5. (4 marks)

A general ellipsoid has semi-axes lengths *a, b* and *c* as shown in the diagram below and has a volume given by $V = \dfrac{4\pi abc}{3}$.



Consider the ellipsoid where the relationship between the semi-axes lengths is that *b* is three times *a*, and that the sum of *a* and *c* is 42 cm.

(a) Show that the volume of this ellipsoid is given by $168\pi a^2 - 4\pi a^3$. [1]

$$V = \frac{4\pi(a)(3a)(42-a)}{3} = \frac{12\pi a^2(42-a)}{3} = \frac{504\pi a^2 - 12\pi a^3}{3} = 168\pi a^2 - 4\pi a^3$$

(b) Use the increments formula to estimate the change in volume of the ellipsoid when *a* increases from 30 cm to 30.5 cm. [3]

$\Delta V \approx 336\pi(30) - 12\pi(30)^2 \times 0.5$

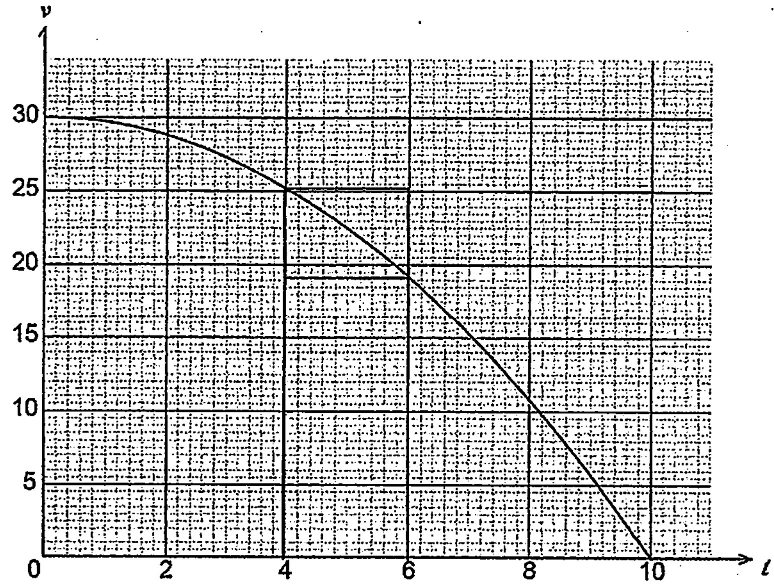
≈ -1130.97

$\therefore$ Decrease of 1130.97 cm^3

6. (6 marks)

A train is travelling at 30 metres per second when the breaks are applied. The velocity of the train is given by the equation $v = 30 - 0.3t^2$ where t represents the time in seconds after the breaks are applied.

The velocity- time graph is shown below.



The area under the velocity-time graph gives the total distance travelled for a particular time period.

(a) Complete the tables below and estimate the distance travelled by the train during the first 6 seconds by calculating the mean of the rectangles (overestimate and underestimate). The rectangles for the 4-6 seconds interval are shown in the table below.

Time (t)	0	2	4	6
Velocity (v)	30	28.8	25.2	19.2

Rectangle	0-2	2-4	4-6	Total
Underestimate	57.6	50.4	38.4	146.4
Overestimate	60	57.4	50.4	168

Estimated distance travelled: 314.4 / 2 = 157.2m [5]

(b) The exact distance travelled is during the first 6 seconds is 158.4m.
How could you determine a better estimate of the distance travelled by the train during the first six seconds than the one determine in (a)?

Use smaller width rectangles

7. (10 marks)

The displacement of a particle moving along a straight line at time t seconds is given by $s = t^3 - 4t^2 + 4t - 10$ metres.

(a) Determine the change in displacement in the first 2 seconds. [2]

$= s(2) - s(0)$

$= 10 - 10$

$= 0m$

(b) Determine the velocity of the particle when $t = 5$ seconds. [2]

$\left.\frac{ds}{dt}\right|_{t=5} = 3t^2 - 8t + 4$

$= 39 m/s$

(c) Determine when the particle is instantaneously at rest. [2]

$0 = 3t^2 - 8t + 4$

$t = 2, \quad t = 2/3$

(d) Determine the initial acceleration of the particle. [2]

$\left.\frac{dV}{dt}\right|_{t=0} = 6t - 8$

$a = -8ms^{-2}$

(e) Determine the distance travelled in the first 2 seconds. [2]

$= \int_0^2 |3x^2 - 8x + 4| \, dx$

$= 2.37m$

8. (6 marks)

A company's revenue each week is $\$(800 + 1000n - 20n^2)$ where n is the number of employees. The company spend \$760 per employee for wages and materials.

(a) Write an expression for the company's weekly profit, P dollars. [1]

$P = (800 - 1000n - 20n^2) - 760n$

$= 800 - 240n - 20n^2$

(b) Determine the number of employees required for maximum profit and hence calculate the maximum weekly profit. [3]

$P' = 240 - 40n$

$0 = 240 - 40n$

$n = 6$ $\therefore$ 6 workers ; profit = \$1520 per week

(c) Determine the marginal revenue when $n = 4$ and explain what this tells us about revenue. [2]

$R'(n) = 1000 - 40n$

$\Delta R = R'(4) \times 1$

$= 1000 - 40(4)$

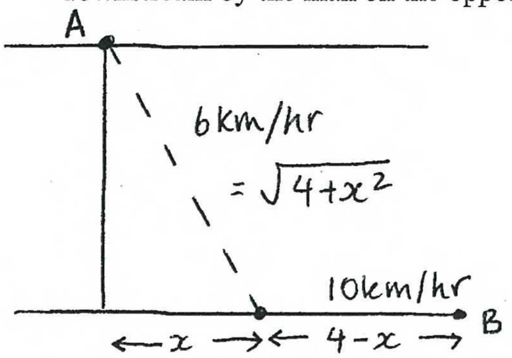
$= \$840$

Marginal revenue is \$840, this is how much extra revenue you expect from the next worker

9. (7 marks)

A man at point A on the bank of a river 2km wide wishes to reach a point B, 4km down from point A on the opposite bank. He can travel in a boat at 6km/hr and ride a bicycle at 10km/hr.

(a) Given that Time = $\frac{\text{Distance}}{\text{Speed}}$, show that the equation for the time taken for the man to reach the point across the river is $t = \frac{\sqrt{4+x^2}}{6} + \frac{4-x}{10}$ where x is the distance landed downstream by the man on the opposite bank. [3]



$$t = \frac{\sqrt{4+x^2}}{6} + \frac{4-x}{10}$$

(b) Determine how far downstream he must land on the opposite bank in order to reach point B in a minimum time and state that minimum time. [4]

$$t' = \frac{1}{12}(4+x^2)^{-1/2} \cdot (2x) - \frac{1}{10}$$

$$0 = \frac{x}{6(4+x^2)^{1/2}} - \frac{1}{10}$$

$$x = 1.5$$

$\therefore$ He must cross to a point 1.5km downstream

min time $= \frac{2}{3}$ hrs.